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Fuel-Optimal Spacecraft Rendezvous with Hybrid On–Off Continuous and Impulsive Thrust

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Nomenclature

J	=	cost function
N	=	number of impulses
N_{LT}	=	number of low-thrust-independent thrusters
p	=	primer-vector magnitude
$\boldsymbol{p}$	=	primer vector
$\boldsymbol{p}_m$	=	primer-vector absolute maximum
$\boldsymbol{r}$	=	relative position vector
$\boldsymbol{r}_j$	=	space collocation of the jth impulse
t_f	=	final time
t_{int}	=	intermediate time
t_j	=	time instant of the jth impulse application
t_m	=	midcourse time corresponding to the primer absolute maximum
t_0	=	initial time
$\boldsymbol{u}$	=	thrust unit vector
$\boldsymbol{V}$	=	relative velocity vector
x, y, z	=	coordinates in the local vertical, local horizontal coordinate system
$\Delta \boldsymbol{V}_j$	=	jth impulse vector
$\delta(t - t_j)$	=	Dirac's delta at time t_j
ϑ	=	anomaly on the circular orbit
$\boldsymbol{\lambda}_r$	=	position adjoint vector
$\boldsymbol{\lambda}_V$	=	velocity adjoint vector
Γ	=	control vector magnitude
$\Phi(t)$	=	state transition matrix for the Clohessy–Wiltshire equations
$\Psi(t)$	=	convolution integral matrix for the Clohessy–Wiltshire equations due to optimal unbounded control
ω_{LVLH}	=	angular velocity of the local vertical, local horizontal coordinate system

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I. Introduction

A HYBRID method is introduced for a minimum-propellant rendezvous and docking maneuver of a spacecraft having both multilevel continuous and impulsive thrusters. We assume that the chaser spacecraft uses a set of independently operated time-continuous low-thrust actuators for the rendezvous maneuvering far from the target and then switches to impulsive thrusters for the docking maneuver.

Several researchers have studied the case of rendezvous and docking maneuvers of spacecraft with continuous thrust [1–6]. In particular, Guelman and Aleshin [4] proposed to find the minimum-propellant solution by imposing an upper saturation on the thrust magnitude, by using Pontryagin's principle and by iteratively changing the initial costate to minimize the error in reaching the final desired condition. Furthermore, a vast amount of literature exists on orbital-change maneuvers with impulsive thrust [7–9]. In particular, Lawden [7] established a set of six necessary conditions of optimality, based on the definition of the primer vector as the adjoint velocity, and Neutstadt [8] demonstrated that for linear systems, the number of impulses is upper-limited by the number of the final conditions.

In our paper, we build upon the previously mentioned references and give the following two original contributions. First, we consider the maneuver divided into two phases: a rendezvous phase with continuous thrust and a final docking phase with impulsive thrust. Second, for the continuous-thrust phase, we assume each of the three thrust components to be generated by a cluster of independent low-thrust engines, each one operating either at the maximum or at zero thrust (on–off). We believe that the main advantage of our approach consists in being practically applicable, in principle, to spacecraft using current solar–electrical limited-power propulsion technology.

An outline of our proposed approach follows. For the first phase of the maneuver (rendezvous), a minimization algorithm is used to find the adjoint initial conditions and the final time that bring the chaser vehicle to the desired final condition of the first phase (intermediate condition of the whole maneuver). In particular, we extend the method proposed in [4] by considering multiple discrete levels of thrust. Furthermore, we limit the thrust component along each direction (x, y, and z) to better represent the reality of engine clusters mounted on the sides of the spacecraft. For the second phase of the maneuver (docking), the two-impulse maneuver is first determined in closed form and the related primer-vector history is analyzed. Then additional impulses are added as needed to satisfy Lawden's conditions by optimizing their time location using a gradient-search technique [10–13]. This procedure is similar to the one previously used for orbital-transfer optimization [10,11].

In the present paper, we adopt the linear Hill–Clohessy–Wiltshire (HCW) dynamic model and we neglect the attitude dynamics.

II. Optimum-Control Problem Definition and Proposed Solution

The problem statement is as follows: given an initial state of the chaser spacecraft with respect to the target-centered local vertical, local horizontal (LVLH) coordinate system, an intermediate state and a maneuver time $t_f - t_0$ determine 1) the optimum continuous-thrust profile for the first phase of the maneuver (from the initial to the intermediate state; in particular, a set of discrete admissible thrust values is assumed) and 2) the optimum sequence of impulses for the second phase of the maneuver (from the intermediate state to the docking condition, that is, zero relative position and velocity of chaser vs target).

The normalized form [4] of the HCW equations is used to represent the relative state-vector evolution:

$$\begin{Bmatrix} \boldsymbol{V} \\ \dot{\boldsymbol{V}} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix} \begin{Bmatrix} \boldsymbol{r} \\ \boldsymbol{V} \end{Bmatrix} + \begin{Bmatrix} 0 \\ \Gamma \boldsymbol{u} \end{Bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

where I represents the 3×3 identity matrix, and the x, y, and z axes are oriented along the opposite direction to the target velocity, the radius vector from Earth to target, and the normal to the orbit plane, respectively.

For the first phase of the maneuver (continuous thrust), the cost function and related Hamiltonian are [4]

$$J = \frac{1}{2} \int_{t_0}^{t_{\mathrm{int}}} (\Gamma \boldsymbol{u})^T \cdot (\Gamma \boldsymbol{u}) \, \mathrm{d}t \tag{2}$$

$$H = \tfrac{1}{2} \Gamma^2 \boldsymbol{u} \cdot \boldsymbol{u} + \boldsymbol{\lambda}_r^T \cdot \boldsymbol{V} + \boldsymbol{\lambda}_V^T \cdot (A_1 \boldsymbol{r} + A_2 \boldsymbol{V} + \Gamma \boldsymbol{u}) \tag{3}$$

By applying the minimum principle, it yields [4]

$$\Gamma \boldsymbol{u} = -\boldsymbol{\lambda}_V \tag{4}$$

Moreover, the time evolution of the costate is described by

$$\begin{Bmatrix} \dot{\boldsymbol{\lambda}}_r \\ \dot{\boldsymbol{\lambda}}_V \end{Bmatrix} = - \begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix}^T \begin{Bmatrix} \boldsymbol{\lambda}_r \\ \boldsymbol{\lambda}_V \end{Bmatrix} \tag{5}$$

Because we consider each thrust component to be generated by a cluster of N_{LT}-independent, continuously operated on–off actuators having, in general, different performances, the acceleration for each direction is constrained to assume a set of discrete values between zero and a maximum. The consequent discretization and saturation introduce a state error at the intermediate-maneuver condition.

The optimum control problem for the first phase of the maneuver is solved by iteratively searching for the values of the initial condition of the costate vector and final time, which minimize the norm of the error between the desired and actual final state of the first phase of the maneuver (intermediate state of the whole maneuver). This error is found by propagating Eq. (1). As an initial guess, the initial costate vector for unbounded continuous control is used [4]:

$$\boldsymbol{\lambda}_0 = -\Psi^{-1}(t_f - t_0) \left[\Phi(t_f - t_0) \begin{Bmatrix} \boldsymbol{r}_0 \\ \boldsymbol{V}_0 \end{Bmatrix} - \begin{Bmatrix} \boldsymbol{r}_f \\ \boldsymbol{V}_f \end{Bmatrix} \right] \tag{6}$$

where $\Psi(t)$ is the convolution integral for the state vector due to the optimal unbounded control (see the appendix of [4] for the explicit form).

In particular, we used a custom implementation of a multicriterion, multivariable routine for direct-search optimization, based on the Hooke–Jeeves algorithm [14]. The Matlab `fsolve` routine, which was used in [4], was not capable of treating the discretized problem.

For the second phase of the maneuver (impulsive thrust), the following cost function is considered:

$$J = \int_{t_{\mathrm{int}}}^{t_f} |\Gamma \boldsymbol{u}| \, \mathrm{d}t = \sum_{i=1}^{N} \Delta V_i, \qquad N = 2, \ldots, 6 \tag{7}$$

where we assumed

$$\Gamma \boldsymbol{u} = \sum_{j=1}^{N} \Delta \boldsymbol{V} \delta(t - t_j)$$

The maximum value of N is six, because of Neustadt's [8] theorem. To solve this optimum control problem related to the second phase of the maneuver, we need to find the optimal combination of the following variables: total number of impulses, time of each impulse t_j, space collocation of each impulse $\boldsymbol{r}_j$, and consequent value of each impulse $\Delta \boldsymbol{V}_j$. This optimum control problem is conveniently translated into the following necessary, and sufficient for linear systems, Lawden's [7] conditions of optimality on the primer vector (costate associated with the velocity $\boldsymbol{\lambda}_V$), which has the same direction as the thrust vector:

Condition 1) p is a continuous function of time up to the first derivative

Condition 2) during coasting ($\Gamma = 0$), $p < 1$

Condition 3) at an impulse $p = 1$, tangent to 1 from below

Condition 4) at an impulse time $\boldsymbol{u} = \boldsymbol{p}$

Condition 5) if $\dot{p}(t_0) \geq 0$, initial coasting is needed

Condition 6) if $\dot{p}(t_f) \leq 0$, final coasting is needed.

Interestingly, final coasting is never needed for the docking case, that is, for zero position and velocity at final time. Indeed, final coasting would imply a nontrivial solution for a six-equation homogeneous system with seven unknowns (time instant t^* and the associated state vector). But the determinant of the corresponding transition matrix never vanishes (at $t^* = t_f$ it gives an identity matrix)

$$\det(\Phi) = \cos^4(t_f - t^*) + 2\sin^2(t_f - t^*)\cos^2(t_f - t^*) + \sin^4(t_f - t^*) \tag{8}$$

Then a final $\Delta \boldsymbol{V}$ is always required for the docking case.

The following algorithmic steps are applied to satisfy Lawden's six conditions and minimize the cost function [10,11]:

Step 1) The two impulses bringing the chaser spacecraft from the intermediate to the final state are calculated in closed form [9].

Step 2) The maxima of the magnitude of the primer vector exceeding one are analytically found for this maneuver.

Step 3) A new midcourse impulse is added at the primer absolute maxima (condition 4).

Step 4) The impulses before and after the new midcourse impulse are modified to match the boundary conditions. The calculation for steps 3 and 4 exploits linear system theory and can be found in [11].

Step 5) The midcourse impulse locations in time and space are adjusted by a conjugate gradient-search technique to force the satisfaction of conditions 1 and 3. Consequently, steps 3 to 4 are iteratively repeated.

Step 6) Repeat steps 2 to 5 for each segment of trajectory included between two impulses. The boundary conditions to be used in step 4 are the current-segment initial and final states.

Step 7) Stop when either the number of impulses reaches six or Lawden's conditions are satisfied.

Table 1 Boundary conditions for the sample maneuver

Variable name	Measurement units	Value
$\boldsymbol{r}(t_0)$	km	$\begin{bmatrix} 15 & 0 & 2 \end{bmatrix}^T$
$\boldsymbol{V}(t_0)$	ms^{-1}	$\begin{bmatrix} 10 & 0 & 2 \end{bmatrix}^T$
$\boldsymbol{r}(t_{\mathrm{int}})$	km	$\begin{bmatrix} -3 & 0 & 0 \end{bmatrix}^T$
$\boldsymbol{V}(t_{\mathrm{int}})$	ms^{-1}	$\begin{bmatrix} 2.5 & 0 & 0 \end{bmatrix}^T$
$\boldsymbol{r}(t_f)$	km	$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$
$\boldsymbol{V}(t_f)$	ms^{-1}	$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$
t_0	s	0
t_f	s	13,000
t_{int}	s	8500

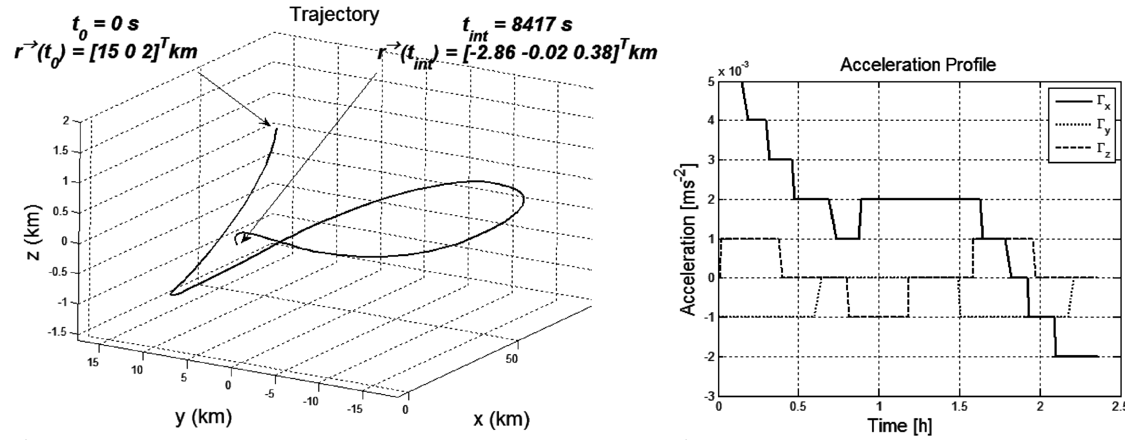

Fig. 1 Results for the maneuver's first phase: a) trajectory obtained and b) continuous acceleration profile.

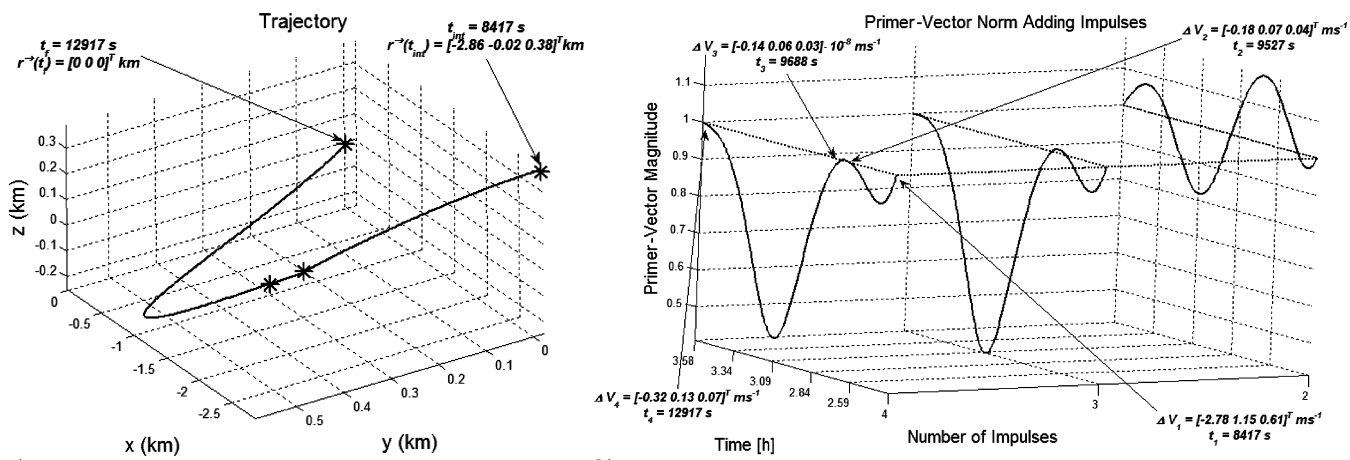

Fig. 2 Results for the maneuver's second phase: a) obtained trajectory and b) primer-vector norm and impulses.

III. Implementation Details and Simulation Results

The preceding algorithms were implemented in Matlab. A sample rendezvous and docking maneuver is presented here with the boundary conditions of Table 1. In particular, for the first phase of the maneuver (continuous thrust), we considered a cluster of five electrical thrusters for each thrust component ($N_{\mathrm{LT}} = 5$), each able to give a fixed level of acceleration (10^{-3} ms^{-2}). The maximum resulting acceleration with all electrical thrusters on is then 5×10^{-3} ms^{-2}.

The target spacecraft orbits at an altitude of 480 km above the Earth's surface. Figure 1 reports the optimal trajectory for the low-thrust segment and the corresponding acceleration profile. The desired switching structure is obtained, which can be followed by a cluster of five different engines. The resulting cost is 22.445 ms^{-1} for the first stage.

The error in reaching the intermediate state, defined as the difference between the desired condition and the actual one, is $[-0.14\ 0.02\ -0.38]^T$ km and $[-4.22\ 2.11\ 5.45]^T \times 10^{-4}$ km $\cdot$ s^{-1}. The intermediate time is slightly adjusted by the algorithm from 8500 to 8417 s.

Figure 2a reports the second-phase optimal trajectory, the stars indicating the space collocation of the impulses. Figure 2b gives a visual representation of the impulse insertion, up to four, and the change of the primer-vector norm. In particular, the initial-guess nonoptimal primer for the two-impulse maneuver is shown. By adding impulses and by applying the conjugate-gradient technique, the final optimum with four impulses is obtained, with impulse magnitudes and burn instants as reported in Fig. 2b.

No initial coasting was needed for the presented case.

The total cost for the maneuver is 23.390 ms^{-1}. To show the potential of the proposed technique, the same initial conditions of Table 1 were used to simulate a rendezvous to the origin of the LVLH frame with the thrust-upper-bounded approach of [4]. By imposing the whole thrust (the norm of the control vector) to be saturated at 5×10^{-3} ms^{-1} and by using a final time of 13,000 s as in the hybrid approach, the solution restitutes a cost of 22.176 ms^{-1}. We can conclude that the hybrid-thrusters method is effective in optimizing spacecraft-rendezvous maneuvers, because it generates a suitable profile for the control variables without significantly increasing the cost with respect to the purely continuous-thrust approach with thrust modulation and gimballing.

We should furthermore underline the fact that, the final time being the same for the two compared approaches, although in our hybrid method, the thrust can assume higher values, it was expected that the new technique restituted a higher cost.

IV. Conclusions

To consider real thrusters for rendezvous and docking, a hybrid technique was presented for minimum-propellant maneuvers. The relative motion between the agents involved in the proximity operations is represented by the Clohessy and Wiltshire linear model. We considered a finite set of low thrusts for distant maneuvers and thrust impulses for the very last phase of flight to the target. The method developed can be applied when a cluster of either different or equal thrusters is mounted on the chaser spacecraft. Therefore, electrical engines can be used for the distant segment and chemical engines for the final stage, in which increased accuracy and control authority is needed. The approach takes into account the real limitations of current space-qualified electrical thrusters, more specifically, the difficulty of varying the thrust magnitude continuously in time.

The low-thrust segment is solved by setting up a minimization problem and by using a first-order algorithm. The second phase is solved by imposing Lawden's [7] conditions via a direct-search technique. In conclusion, we generated a reliable procedure to determine the switching structure of the low-thrust part and the number, magnitudes, directions, times, and space collocation of the impulses for the second phase of the maneuver.

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